MNYMO

August 22, 2026

General

The answer key is below.

p	Answer	p	Answer
1	128	11	$-\frac{3}{4}$
2	280	12	13
3	22	13	241
4	6	14	48
5	15	15	239
6	$\frac{47}{3}$	16	$\frac{\sqrt{3}}{2}$
7	18	17	18
8	1024	18	$\frac{8}{3333}$
9	67	19	$\sqrt{13}-2$
10	6076	20	59049

1. On Monday, there is one slime-duck in a lake at noon. Every night, each slime-duck splits into two slime-ducks, which themselves split in the follwoing nights. How many slime-ducks will be in the lake after seven nights have passed?

   Proposed by: Zack Berchenko

   Answer: $\boxed{128}$

   Solution: As the number of slime-ducks doubles every night for seven nights, the answer is $2^7 = \boxed{128}$.

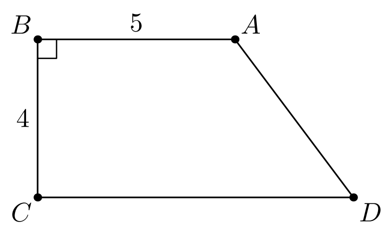
2. My locker combination contains three numbers between 1 and 50, inclusive. The first number is a perfect square, the second number is divisible by 5, and the third number is a power of 3. Compute the number of possible locker combinations. (The numbers in the combination don't need to be distinct.)

   Proposed by: Taison Scofield

   Answer: $\boxed{280}$

   Solution: We have 7 choices for the first number, 10 choices for the second number, and 4 choices for the third number for a total of $7 \cdot 10 \cdot 4 = \boxed{280}$ possible combinations.

3. Trapezoid $ABCD$ has $AB \parallel CD$ and $\angle ABC = 90^\circ$. If $AB = 5$, $BC = 4$, and $[ABCD]$ is 26, compute the perimeter of $ABCD$. (Note: $[ABCD]$ denotes the area of polygon $ABCD$.)

   Proposed by: Samuel Kretzschmar

   Answer: $\boxed{22}$

   Solution: From the trapezoid area formula, $26 = 4 \cdot \frac{5+CD}{2}$, so $13 = 5 + CD$ and $CD = 8$. Let E be the foot of the altitude from A to CD. We have $CE = 5$ so $ED = 8 - 5 = 3$. By the Pythagorean Theorem, $AD = 5$. Thus, our final answer is $5 + 5 + 8 + 4 = \boxed{22}$.

4. Three years ago, Bob's age was exactly twice that of Alice's, and Alice's age was exactly the square of Caleb's age. Given that Alice is currently exactly 6 or 7 years old, how many years older than Caleb is Bob?

   Proposed by: Zack Berchenko

   Answer: $\boxed{6}$

   Solution: Let a be Alice's age. Then $a - 3$ is a square, so $a = 7$. Thus, three years ago, Bob was 8 years old and Caleb was 2 years old. Thus, the age difference is $\boxed{6}$.

5. Compute the minimum number of 9×15 rectangular tiles needed to form a square.

   Proposed by: Taison Scofield

   Answer: $\boxed{15}$

Solution: The square must have some integer side length s, so s^2 is a multiple of $9 \cdot 15 = 3^3 \cdot 5$, implying s must have 3^2 and 5 as factors. Therefore, s must be at least 45. This is achievable by forming a 3×5 grid of these tiles, which requires $\boxed{15}$ tiles.

6. Compute the average of the lengths of the three altitudes of a 15–20–25 right triangle.

   Proposed by: Taison Scofield

   Answer: $\boxed{\frac{47}{3}}$

   Solution: The altitudes have lengths 15, 20, and $\frac{15 \cdot 20}{25} = 12$, so our answer is $\frac{15+20+12}{3} = \boxed{\frac{47}{3}}$.

7. Compute the number of ways to choose a positive number of squares from the 2×3 grid below such that the squares together form a rectangle.

   Proposed by: Samuel Kretzschmar

   Answer: $\boxed{18}$

   Solution: We can consider our choices of vertical and horizontal edges separately. There are $\binom{3}{2}$ ways to choose the horizontal edges and $\binom{4}{2}$ ways to choose the vertical edges, yielding a final answer of $3 \cdot 6 = \boxed{18}$.

8. Zack and Jack are playing the 2^n game! In this game, Zack jumps once, then Jack jumps twice, then Zack jumps four times, then Jack jumps eight times. This continues until someone gives up, with each player jumping twice as much as the previous person on each turn. After some positive number of turns, Jack gives up. If the total number of times Zack has jumped is a multiple of 13, then what is the minimum number of times Zack jumped on his last turn?

   Proposed by: Zack Berchenko

   Answer: $\boxed{1024}$

   Solution: The condition is equivalent to $13 \mid 1+4^1+\cdots+4^n = \frac{4^{n+1}-1}{3}$, so $4^{n+1} \equiv 1 \pmod{13}$. Since $4^3 \equiv 64 \equiv -1 \pmod{13}$, the minimum such $n+1$ is 6. Thus, the smallest possible number of times Zack jumped on his last turn is $4^5 = \boxed{1024}$.

9. Daniel has a fair 100-sided die with faces labeled 1, 2, ..., 100. Compute the expected value of the square of Daniel's roll divided by the expected value of Daniel's roll.

   Proposed by: Michael Luo

   Answer: $\boxed{67}$

   Solution: Let $n = 100$. The answer is

$$\frac{\frac{1^2+2^2+\cdots+n^2}{n}}{\frac{1+2+\cdots+n}{n}} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3} = \boxed{67}.$$

10. Michael is playing a number game. He starts with an integer and repeatedly applies the following rule until he reaches 1.

    - If his current number is not divisible by 3, he subtracts 1 from it.
    - If his current number is divisible by 3, he divides it by 3.

    If Michael starts the game with $3^{2026}-1$, he will eventually reach 1. How many moves will it take him to reach 1? (For example, if $n=2$, it takes 1 move to reach 1.)

    Proposed by: Taison Scofield

    Answer: $\boxed{6076}$

    Solution:

    Claim. Each move either decreases the last digit of n in base 3 by 1, or removes a trailing digit of 0.

    Proof. Subtracting 1 (when $3 \nmid n$) decreases the last base-3 digit by 1, and dividing by 3 (when $3 \mid n$) removes a trailing 0 digit. □

    Since we stop once we reach $n=1$, the leading digit only needs to be brought down to 1, not all the way to 0. Therefore, if n has k digits in base 3 with digit sum $S_3(n)$, it takes $S_3(n)+k-2$ moves to reach 1.

    Since $3^{2026}-1$ consists of 2026 copies of the digit 2 in base 3, we have $S_3(n) = 2 \cdot 2026$ and $k = 2026$. This results in a total of $2(2026)+2026-2 = \boxed{6076}$ moves.

    Solution 2:

    Claim. It takes 3 steps to go from $3^{n+1}-1$ to 3^n-1 for $n \geq 1$.

    Proof. We go $3^{n+1}-1 \to 3^{n+1}-2 \to 3^{n+1}-3 \to 3^n-1$. □

    Thus, it will take $3 \cdot (2026-1) = 6075$ steps to go from $3^{2026}-1$ to $3^1-1=2$. We have one additional step after that to go to 1, for a total of $\boxed{6076}$ steps.

11. Jeff is given some angle θ and is asked to compute $(2^{\cos(\theta)})^2(2^{\sin(\theta)})^2$. Unfortunately, he simplifies this as $2^{\cos^2(\theta)}2^{\sin^2(\theta)}$, and proceeds correctly from there. He is surprised to get the right answer. Compute $\sin(2\theta)$.

    Proposed by: Jefferson Zhou

    Answer: $\boxed{-\frac{3}{4}}$

    Solution: The correct expression is $2^{2\cos\theta+2\sin\theta}$, while the incorrect expression is $2^{\cos^2\theta+\sin^2\theta} = 2^1$. Since these expressions are equal, $2\cos\theta+2\sin\theta = 1$, so

    $$1+\sin 2\theta = \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = (\sin\theta+\cos\theta)^2 = \frac{1}{4},$$

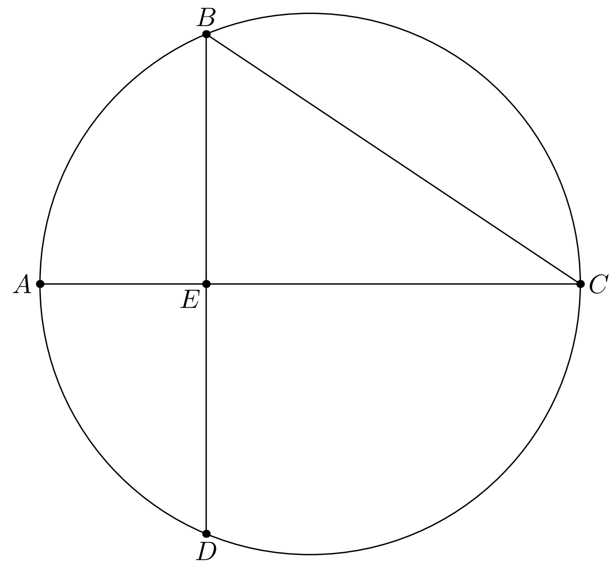
    so $\sin 2\theta = \boxed{-\frac{3}{4}}$.

12. Let AC be a diameter of circle ω and BD be a chord on ω such that $AC \perp BD$. Let E be the point where AC and BD intersect. Given that $AE=8$ and $BC = 6\sqrt{13}$, compute the radius of ω.

    Proposed by: Taison Scofield

Answer: $\boxed{13}$

Solution:



Let $CE = x$ and $DE = EB = y$. By Power of a Point, we have $8x = y^2$.

Also, since $EB \perp EC$, we have $x^2 + y^2 = BC^2 = 468$. Substituting $8x$ for y^2 yields $x^2 + 8x - 468 = 0$. By the quadratic formula, the positive solution is $x = 18$.

Since AC is the diameter, the radius of ω is $\frac{8+18}{2} = \boxed{13}$.

13. Let x be a real number such that

$$x^2 + x + \frac{1}{x} + \frac{1}{x^2} = 10.$$

Compute the sum of all possible values of $x^4 + \frac{1}{x^4}$.

Proposed by: Jefferson Zhou

Answer: $\boxed{241}$

Solution: Let $a = x + \frac{1}{x}$. Then, $a^2 - 2 = x^2 + \frac{1}{x^2}$, so $a^2 - a - 12 = (a-4)(a+3) = 0$. Observe that both $x + \frac{1}{x} = -4$ and $x + \frac{1}{x} = 3$ have real solutions. In the former case, $x^2 + \frac{1}{x^2} = 14$, so $x^4 + \frac{1}{x^4} = 14^2 - 2 = 194$, and in the latter case, $x^2 + \frac{1}{x^2} = 7$, so $x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$. Thus, the answer is $194 + 47 = \boxed{241}$.

14. A positive integer is *kretzschmary* if it cannot be written as the sum of two positive composite numbers. Compute the sum of all kretzschmary positive integers.

Proposed by: Samuel Kretzschmar

Answer: $\boxed{48}$

Solution: As the smallest composite number is 4, all integers from 1 through 7 are kretzschmary, but $8 = 4 + 4$ is not. For a kretzschmary integer $n > 8$, observe that $n - 4$, $n - 6$, and $n - 8$ must all be not composite; however, they are distinct modulo 3, so the one divisible by 3 must be equal to 3 itself. This yields $n = 7$, $n = 9$, or $n = 11$; we already saw 7 is kretzschmary, and 9 and 11 can be shown to be kretzschmary as well.

Thus, the final answer is

$$(1 + 2 + \cdots + 7) + 9 + 11 = 28 + 20 = \boxed{48}.$$

15. How many ways are there to color a 2×6 grid of squares red or blue such that no two red squares share an edge?

    Proposed by: Taison Scofield

    Answer: $\boxed{239}$

    Solution: We can use recursion, starting from the leftmost column and working our way to the right. There are three possible cases:

    - A: both the top and bottom tiles are blue; the column to the right can be cases A, B, or C
    - B: the top tile is red and the bottom tile is blue; the column to the right can be cases A or C
    - C: the top tile is blue and the bottom tile is red; the column to the right can be cases A or B

    Let (A, B, C) represent the amount of possibilities for each state for a $2 \times n$ grid.

    - $n = 1$: $(1, 1, 1)$
    - $n = 2$: $(1 + 1 + 1, 1 + 1, 1 + 1) = (3, 2, 2)$
    - $n = 3$: $(3 + 2 + 2, 3 + 2, 3 + 2) = (7, 5, 5)$
    - $n = 4$: $(7 + 5 + 5, 7 + 5, 7 + 5) = (17, 12, 12)$
    - $n = 5$: $(17 + 12 + 12, 17 + 12, 17 + 12) = (41, 29, 29)$
    - $n = 6$: $(41 + 29 + 29, 41 + 29, 41 + 29) = (99, 70, 70)$

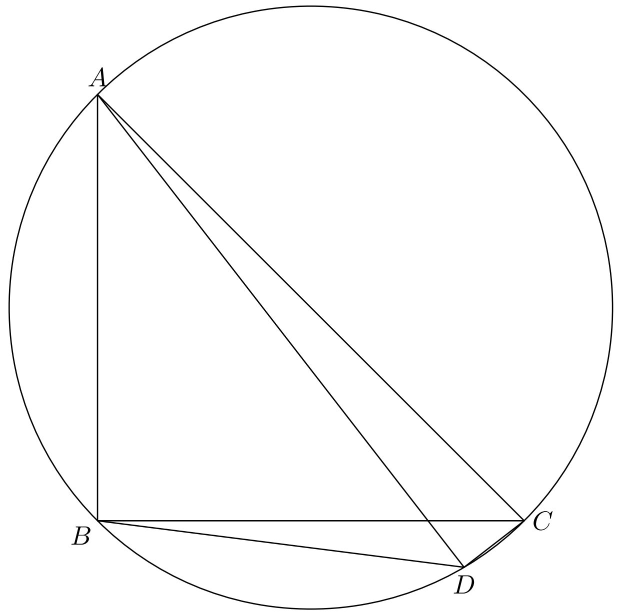
    Adding the values of A, B, and C, we obtain $\boxed{239}$ total ways to color the grid under the given constraints.

16. Triangle ABC with circumcircle ω satisfies $AB = BC = 1$ and $\angle BAC = 45°$. Point D lies on minor arc BC of ω such that $[ACD] = \frac{1}{8}$. Compute BD.

    Proposed by: Jefferson Zhou

    Answer: $\boxed{\frac{\sqrt{3}}{2}}$

    Solution:



Since $ABDC$ is cyclic, $\angle ADC = \angle ABC = 90°$. Thus, $\frac{1}{8} = [ACD] = \frac{AD \cdot CD}{2}$, so $AD \cdot CD = \frac{1}{4}$. Also, by the Pythagorean Theorem, $AD^2 + CD^2 = AC^2 = 2$.

Now, by Ptolemy,

$$BD \cdot AC + AB \cdot DC = BD\sqrt{2} + CD = AD \cdot BC = AD,$$

so $BD\sqrt{2} = AD - CD$. Now,

$$2BD^2 = (AD - CD)^2 = AD^2 + CD^2 - 2AD \cdot CD = 2 - \frac{1}{2} = \frac{3}{2},$$

so $BD^2 = \frac{3}{4}$, and $BD = \boxed{\frac{\sqrt{3}}{2}}$.

17. Andy the Ant starts at the lattice point $(0, 0)$ and wants to reach $(6, 7)$. He may move one unit up, down, or right at each step. The bug must remain within the rectangle $0 \le x \le 6$ and $0 \le y \le 7$, and it may not visit the same lattice point more than once.

    Let N be the number of possible paths Andy can take. Compute $\lfloor \log_2 N \rfloor$.

    Proposed by: Taison Scofield

    Answer: $\boxed{18}$

    Solution: Since Andy cannot move to the left, a column stays the same once he leaves it, so we may consider each column separately. Suppose Andy is currently in column $x = k$. Since he cannot return to a previous lattice point, his vertical movement can only be in one direction. Hence, he may choose any height from 0 to 7 to leave and move to the next column. This yields 8 options per column and 6 columns to leave from, so $N = 8^6 = 2^{18}$. Therefore, $\lfloor \log_2 N \rfloor = \boxed{18}$.

18. Evaluate the summation

$$\sum_{n=1}^{\infty}(0.\underbrace{6\ldots6}_{2n-1\text{ 6's}}\underbrace{7\ldots7}_{2n-1\text{ 7's}} - 0.\underbrace{6\ldots6}_{2n\text{ 6's}}\underbrace{7\ldots7}_{2n\text{ 7's}}) = (0.67 - 0.6677) + (0.666777 - 0.66667777) + \ldots$$

as a fraction.

Proposed by: Samuel Kretzschmar

Answer: $\boxed{\frac{8}{3333}}$

Solution: We find that

$$\begin{aligned}
0.\underbrace{6\ldots6}_{2n-1\text{ 6's}}\underbrace{7\ldots7}_{2n-1\text{ 7's}} - 0.\underbrace{6\ldots6}_{2n\text{ 6's}}\underbrace{7\ldots7}_{2n\text{ 7's}} &= (0.\underbrace{6\ldots6}_{2n-1\text{ 6's}}7 + 0.\underbrace{0\ldots0}_{2n\text{ 0's}}\underbrace{7\ldots7}_{2n-2\text{ 7's}}) - (0.\underbrace{6\ldots6}_{2n\text{ 6's}} + 0.\underbrace{0\ldots0}_{2n\text{ 0's}}\underbrace{7\ldots7}_{2n\text{ 7's}}) \\
&= (0.\underbrace{6\ldots6}_{2n-1\text{ 6's}}7 - 0.\underbrace{6\ldots66}_{2n\text{ 6's}}) + (0.\underbrace{0\ldots0}_{2n\text{ 0's}}\underbrace{7\ldots7}_{2n-2\text{ 7's}} - 0.\underbrace{0\ldots0}_{2n\text{ 0's}}\underbrace{7\ldots777}_{2n\text{ 7's}}) \\
&= \frac{1}{10^{2n}} - \frac{77}{10^{4n}}.
\end{aligned}$$

This argument is perhaps easier to see with an example. Consider the case where $n = 3$; then

$$\begin{aligned}
0.6666677777 - 0.666666777777 &= (0.666667 + 0.0000007777) - (0.666666 + 0.000000777777) \\
&= (0.666667 - 0.666666) + (0.0000007777 - 0.000000777777) \\
&= 0.000001 - 0.000000000077 = \frac{1}{10^6} - \frac{77}{10^{12}}.
\end{aligned}$$

Therefore the answer is

$$\sum_{n=1}^{\infty}(0.01^n - 77 \cdot 0.0001^n) = \frac{0.01}{1-0.01} - 77 \cdot \frac{0.0001}{1-0.0001} = \frac{1}{99} - \frac{77}{9999} = \frac{101-77}{9999} = \boxed{\frac{8}{3333}}.$$

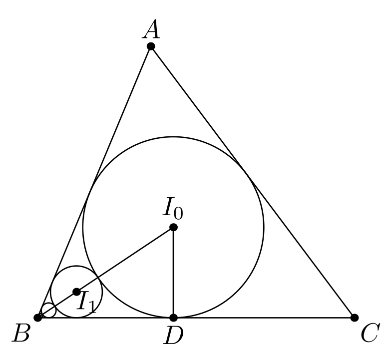
19. In triangle ABC, with $AB = 13, BC = 14$, and $CA = 15$, incircle ω_0 is inscribed, with radius r_0. An infinite sequence of circles $(\omega_n)_{n\geq 1}$ is drawn such that for all $n \geq 1$, ω_n is tangent to AB, BC, and ω_{n-1}, and ω_n is smaller than ω_{n-1}. If r_n denotes the radius of ω_n, compute

$$\sum_{n=1}^{\infty} r_n.$$

Proposed by: Taison Scofield

Answer: $\boxed{\sqrt{13}-2}$

Solution:



First, use $r = \frac{A}{s}$ to find r_0. The semiperimeter is $s = \dfrac{13+14+15}{2} = 21$ and the area equals

$$A = \sqrt{(21)(21-13)(21-14)(21-15)} = \sqrt{(21)(8)(7)(6)} = 84$$

by Heron's, so $r_0 = \frac{84}{21} = 4$.

Let the center of r_0 (also the incenter of the triangle) be point I_0. Notice that each ω_n is tangent to ω_{n-1} along segment BI_0, since both circles' centers lie on the bisector of $\angle B$, and the circles shrink toward B as n grows toward infinity. So BI_0 decomposes as the radius of ω_0 (from I_0 to where ω_1 touches it), plus the diameter of each subsequent circle (since ω_n sits between ω_{n-1} and ω_{n+1}, its full diameter lies along BI_0). In other words,

$$BI_0 = r_0 + 2\sum_{n=1}^{\infty} r_n \implies \frac{BI_0 - 4}{2} = \sum_{n=1}^{\infty} r_n.$$

Let D be the foot of the altitude from I_0 to BC. It's well known that $BD = s - b = 21 - 15 = 6$, so $BI_0 = \sqrt{6^2 + 4^2} = 2\sqrt{13}$.

Putting everything together, we get

$$\sum_{n=1}^{\infty} r_n = \frac{2\sqrt{13} - 4}{2} = \boxed{\sqrt{13} - 2}.$$

20. How many pairs of integers (n, k) with $0 \le k \le n \le 1023$ are there for which $\binom{n}{k}$ is odd?

Proposed by: Samuel Kretzschmar

Answer: $\boxed{59049}$

Solution: Note: For a positive integer n, $\nu_2(n)$ is the largest positive integer k for which $2^k \mid n!$.

Write $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. We require the ν_2 of this to be 0, or $\nu_2(n!) = \nu_2(k!) + \nu_2((n-k)!)$.

Let $s_2(n)$ to be the sum of the digits of n in base 2. By Kummer's, $\nu_2(n!) = n - s_2(n)$, so the equation becomes $n - s_2(n) = k - s_2(k) + n - k - s_2(n-k)$, or $s_2(k) + s_2(n-k) = s_2(n)$.

To achieve this, we must have no carries when adding k and $n-k$ in base 2. We can find the number of solutions in the following manner:

Observe that $1023 = 1111111111_2$. Consider the units digit of k and $n-k$. We can choose both of them to be 0, or exactly one of them to be 1; if both of them were 1, then we would have a carry. Thus, there are 3 ways to choose the units digit of k and $n-k$. For each of the other 9 of the last 10 digits of k and $n-k$ in binary, we have an identical choice, so the total number of ways to choose $(k, n-k)$ is $3^{10} = \boxed{59049}$.