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1. [**5**] An arithmetic sequence $a_1, a_2, a_3, \ldots$ satisfies $a_{20} = 26$ and $a_{26} = 20$. Compute a_{2026}.

2. [**5**] At the ARML lunch, Jack has 12 apples and a deck of 60 cards. The deck has 50 cards worth 2 points and 10 cards worth 5 points. He plays the game "Apples to Apples" and randomly assigns a different card to each of his apples. Compute the expected total points of the cards assigned to Jack's apples.

3. [**5**] Find the positive integer b such that $201_b = 243_5$.

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4. [**6**] In the town Xookland of 100 people, the GDW (Gross Domestic Water) initially starts at 0. Anytime someone receives ℓ liters of water, the GDW increases by ℓ. For example, if one person mysteriously receives 100 liters of water, then gives 50 liters to their neighbor, the GDW increases by 150.

   One morning, one of the people in the town mysteriously receives 100 liters of water. The following process repeats until everyone receives some water: the last person to receive water keeps $\frac{1}{5}$ of it and gives the remainder to another person who has not yet received any water. Afterwards, what will Xookland's GDW be? Round your answer to the nearest integer.

5. [**6**] Rectangle $ABCD$ has side lengths $AB = 28$ and $BC = 16$. Points E and F are constructed such that $ABCD$ is similar to $ABEF$ and E does not lie on CD. Compute the sum of all possible values of DF.

6. [**6**] Every day for 2026 days, Trisha walks in a grove of 2026 trees, labeled Tree 1 to Tree 2026. On day N for $1 \le N \le 2026$, Trisha knocks on every Tree M for which M is a factor of N. At the end of the 2026th day, Tree K has not been knocked on for longer than any other tree. Find K.

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7. [**7**] The fraction $\frac{1}{2026!}$ can be written as a decimal $0.a_1a_2\ldots a_k\overline{b_1b_2\ldots b_n}$ using the $n+k$ digits $a_1, \ldots, a_k, b_1, \ldots, b_n$. What is the smallest possible value of k?

8. [**7**] Jeff has a square piece of bread of side length 1. At each vertex, there is an air bubble in the shape of a quarter circle of radius r centered at that vertex. At two opposite corners of the bread, Jeff cuts out the smallest isosceles right triangle that contains the air bubble and has a vertex at the corner. After this, Jeff is surprised to learn that no part of the other two air bubbles has been cut out. Compute the maximum possible value of r.

9. [**7**] Michael has three fair four-sided dice, each of which has faces labeled 1, 2, 3, and 4. Compute the probability that the sum of the squares of the three rolls is divisible by 3.

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10. [**9**] Consider the sequence $1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, \ldots$, where the number of 2's between consecutive 1's increases by 1 each time. If a_n represents the nth term, find the sum of all positive integers $n \leq 100$ such that $a_n = 2$.

11. [**9**] Let $\triangle ABC$ be an equilateral triangle with a side length of 10. Let G be a point in the interior of $\triangle ABC$. Points D, E, and F are on line segments BC, AC, and AB, respectively, such that $GD \perp BC$, $GE \perp AC$, and $GF \perp AB$. If $GD : GE : GF = 1 : 2 : 2$, compute $AF + AE$.

12. [**9**] In the game 2048, tiles with the same number can be merged to form a tile with twice the value. Each time two tiles merge, your score increases by the value of the newly created tile. Suppose all new tiles that appear are 2s and your score starts at 0. You have just created a 2048 tile, and immediately afterward a new 2 tile appears. If the board now contains only these two tiles, what is your score?

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13. [**10**] Find the unique four-digit integer which is 6 less than a cube and 7 more than a square.

14. [**10**] Sam plays a game on a five-by-five grid that initially has some positive number of coins in the center square. On each turn, he can take any two coins (possibly on different squares) and move them in opposite directions. However, Sam is not allowed to do any moves that would move coins outside the grid. Later in the game, he notices that if he draws a line between the top-left and bottom-right corners of the grid, exactly 2026 coins would be above this line. What is the maximum number of coins below this line?

15. [**10**] If x and y are real and $\log_y x + \log_x y = \log_2 x + \log_2 y = \log_x 4 + \log_y 4 = C$, compute C.

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16. [**12**] Suppose $\triangle ABC$ has side lengths $AB = 13$, $BC = 14$, and $CA = 15$. If M is the midpoint of BC, H_B is the orthocenter of $\triangle BAM$, and H_C is the orthocenter of $\triangle CAM$, find the length of $H_B H_C$.

17. [**12**] Sinner and Alcaraz are playing American ping-pong. Each player has a $\frac{1}{2}$ chance of winning each point, and each point is independent. The players play points until one of the players has at least four points and is beating the other player by at least two points, at which point that player wins the game.

    Alcaraz wins the first point to make the score 1–0. Compute the probability that Alcaraz wins the game.

18. [**12**] Find the unique positive value of x for which

$$\sqrt{x^2 - 7x + 49} + \sqrt{x^2 - 8x + 64} = 13.$$

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19. [**14**] Find the largest possible value of $\gcd(n^3+3, n^5+5)$ across all positive integers n.

20. [**14**] In triangle ABC, $\angle ABC = 90^\circ + \angle ACB$, point M is the midpoint of BC, and D is the intersection of AM with the circumcircle of ABC. If the circumradius of ABC is 15 and $AM = 18$, then what is AD?

21. [**14**] A baker has a cookie in the shape of a regular pentagon. To decorate it, the baker makes n random cuts across the cookie, one at a time, as follows. The baker chooses two distinct sides of the cookie uniformly at random among the 10 possible pairs, then picks two points chosen independently and uniformly at random on these sides. The baker makes the cut connecting these two points.

    Let n be the least integer greater than 1 such that the expected number of pieces into which the cookie is cut is an integer, and let E be that expected number of pieces. Find $n+E$.

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22. [**17**] Real numbers b and c satisfy the system of equations

$$\frac{9b^3+9c^3+1}{15} = b^2 - bc + c^2$$
$$(b-2c)^2 + (c-2b)^2 = \frac{2}{9}.$$

    Compute the sum of all the possible values of bc.

23. [**17**] Two points X and Y are chosen uniformly at random on the perimeter of equilateral $\triangle ABC$ with $AB = 10$. Given that the probability that $XY > 5$ is equal to $\frac{a}{b} - \frac{\pi}{\sqrt{c}}$ for positive integers a, b, and c with $\gcd(a,b) = 1$, find $a+b+c$.

24. [**17**] A right hexagonal prism has side length 6 and height of 4. The plane of the top face is rotated downwards by 30° around one of edges to form a new plane $\mathcal{P}$, which cuts the hexagonal prism into two parts. Compute the portion of $\mathcal{P}$ inside the prism.

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25. [**20**] The 2026 MN Youth Math Outreach staff team consists of Angie Huang, Michael Luo, Austin Wang, Jefferson Zhou, Taison Scofield, Zack Berchenko, Sneha Kundu, Samuel Kretzschmar, Eric Ding, Addison Shi, Andreas Walexon, Jack Gao, Allison Zhang, and Ansh Singh, who together attended some set of high schools in the 2025–26 school year. Estimate the area in square miles of the convex hull formed by these schools.

    Submit a positive real number E. If the correct answer is A, you will earn $\lfloor 20.99 \cdot 2^{-16(1-E/A)^2} \rfloor$ points.

    (The *convex hull* of a finite set of points $\mathcal{P}$ is the smallest convex polygon which contains all points in $\mathcal{P}$. It may be visualized as a rubber band stretched around $\mathcal{P}$.)

26. [**20**] Samuel Jesse Kretzschmar has recently turned 18. He found a very profitable slot machine at a casino, which is free to play and pays him 2^n dollars with probability $\frac{1}{2^n}$ for all integers $n \geq 1$. Estimate the expected number of times he must play before his profit first reaches 10^6 dollars.

    Submit a positive real number E. If the correct answer is A, you will earn $\lfloor 20.99 \cdot 2^{-32(1-E/A)^2} \rfloor$ points.

27. [**20**] A prime is *mnymoy* if it can be written as the concatenation of n and $n+1$ (in that order) for some positive integer n such that either n or $n+1$ is prime. For example, 67 is mnymoy, since it is the concatenation of 6 and 7, and 7 is prime. Compute the number of mnymoy integers at most 10^{10}.

    Submit a positive real number E. If the correct answer is A, you will earn $\lfloor 20.99 \cdot 2^{-32(1-E/A)^2} \rfloor$ points.