# MNYMO

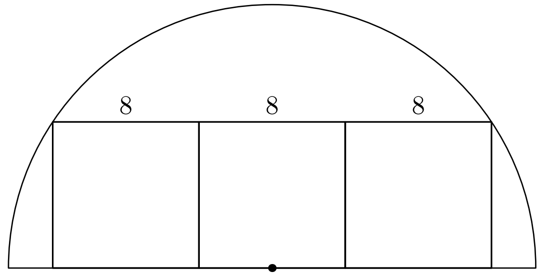
**August 22, 2026**

## Team

The answer key is below.

| p | Answer | p | Answer |
|---|---|---|---|
| 1 | $104\pi$ | 6 | $\frac{23}{72}$ |
| 2 | 23 | 7 | 151200 |
| 3 | 5 | 8 | 16644 |
| 4 | 1029 | 9 | 65 |
| 5 | 32 | 10 | 81 |

1. A semicircle circumscribes 3 adjacent squares of side length 8 as shown in the diagram below. Find the area of the semicircle.




*Proposed by: Samuel Kretzschmar*

**Answer:** $\boxed{104\pi}$

**Solution:** Using the Pythagorean Theorem, we find the radius of the semicircle to be $\sqrt{8^2 + (8+4)^2} = \sqrt{208}$, so the semicircle's area is $\frac{1}{2}\pi(\sqrt{208})^2 = \boxed{104\pi}$.

2. Alice, Bob, and Carol want to paint a house in exactly one hour. If Alice and Bob work together to paint the house, they could finish it in 3 hours. If Alice and Carol work together, they could finish it in 4.5 hours. If Bob and Carol work together, they could finish it in 6 hours.

   Unfortunately, Alice, Bob, and Carol are way too slow to paint the house in only one hour, so Carol recruits some friends who paint at the exact same rate as her. How many friends must Carol recruit so that they can finish the house in one hour?

   *Proposed by: Taison Scofield*

   **Answer:** $\boxed{23}$

   **Solution:** Let Alice, Bob, and Carol's painting rates be $a$, $b$, and $c$ houses per hour. We have

$$a + b = \frac{1}{3}, \quad a + c = \frac{2}{9}, \quad b + c = \frac{1}{6}.$$

   Adding the first two equations and subtracting the third yields $2a = \frac{7}{18}$, so $a = \frac{7}{36}$. We can use this to figure out that $b = \frac{12}{36} - \frac{7}{36} = \frac{5}{36}$ and $c = \frac{8}{36} - \frac{7}{36} = \frac{1}{36}$. Therefore, the three of them can paint together at a rate of $\frac{7}{36} + \frac{5}{36} + \frac{1}{36} = \frac{13}{36}$ houses per hour.

   Each of Carol's friends paints at a rate of $\frac{1}{36}$ houses per hour, so she needs to recruit

$$\frac{1 - \frac{13}{36}}{\frac{1}{36}} = \frac{\left(\frac{23}{36}\right)}{\left(\frac{1}{36}\right)} = \boxed{23}$$

   friends for them to finish in 1 hour.

3. Call an arithmetic sequence of primes *banned term* if it has common difference 6. Find the maximum possible length of a banned term sequence of primes.

   *Proposed by: Jefferson Zhou*

   **Answer:** $\boxed{5}$

   **Solution:** Note that for any prime $p$, the numbers $p$, $p+6$, $p+12$, $p+18$, $p+24$ are distinct modulo 5, so one of them is divisible by 5. Thus, if they are all prime, one of them must be equal to 5, meaning $p = 5$ by size; in this case, the numbers become 5, 11, 17, 23, 29, which are indeed all prime. We clearly cannot have a sequence of length greater than 5, because then the all terms except the last four would have to be 5 by the above result, so the answer is $\boxed{5}$.

4. In the $3 \times 3$ grid below, place nonzero digits in each cell such that the three-digit numbers formed by reading each row right-to-left and each column top-to-bottom conform to the following rules:

   - Row 1: A square Fibonacci number
   - Row 2: A perfect cube
   - Row 3: A number with strictly decreasing digits
   - Column 1: An odd composite number
   - Column 2: A multiple of 37
   - Column 3: A multiple of Row 1

   Compute the sum of the three-digit numbers formed by the rows.

   | | Col 1 | Col 2 | Col 3 |
   |---|---|---|---|
   | Row 1 | | | |
   | Row 2 | | | |
   | Row 3 | | | |

   *Proposed by: Taison Scofield*

   **Answer:** $\boxed{1029}$

   **Solution:** We can fill out the grid in the following order.

   - Row 1: The only 3-digit number that is both a Fibonacci number and a perfect square is 144 ($12^2 = 144$).
   - Column 3: The only 3-digit multiple of 144 that begins with 4 is 432.
   - Row 2: The only 3-digit cube that ends with a 3 is 343.
   - Column 2: The only 3-digit multiple of 37 that begins in 44 is 444.
   - Bottom-left cell: The only number that satisfies the conditions that $130 + x$ is composite, $x$ is odd, and $x > 4$ is $x = 5$.

   Our completed grid is

   | | Col 1 | Col 2 | Col 3 |
   |---|---|---|---|
   | Row 1 | 1 | 4 | 4 |
   | Row 2 | 3 | 4 | 3 |
   | Row 3 | 5 | 4 | 2 |

   and the answer is $144 + 343 + 542 = \boxed{1029}$.

5. You're playing a game that involves repeatedly rolling a fair 12-sided die that has the integers from 1 to 12 on it. You start with zero points, and on each turn you may either roll the dice, or end the game. If you roll a 6 or a 7, your score becomes 0 and the game ends. Otherwise, the number you roll is added to your score. To maximize the expected value of your score, there exists an integer $N$ for which you should always roll again if your score is less than or equal to $N$, but always should stop rolling if your score is more than $N$. Compute $N$.

   *Proposed by: Taison Scofield*

   **Answer:** $\boxed{32}$

   **Solution:** If your score is $x$, the expected number of points from a roll is

$$\frac{1+2+3+4+5+8+9+10+11+12-2x}{12} = \frac{65-2x}{12}.$$

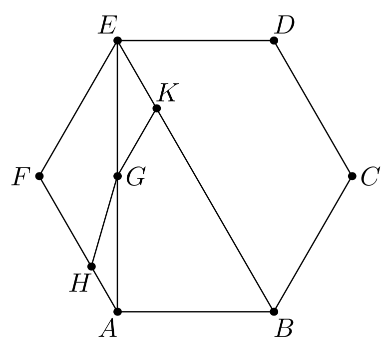
   This is positive for $x < \frac{65}{2}$ and negative for $x > \frac{65}{2}$. Thus the expected value of points increases when $x \leq 32$ and decreases when $x \geq 33$; thus the only answer is $N = \boxed{32}$.

6. Let $ABCDEF$ be a regular hexagon. Let $G$ lie on line segment $AE$ such that $AG = EG$. Let $H$ lie on line segment $AF$ such that $2AH = FH$. Let $K$ lie on line segment $BE$ such that $3EK = BK$. Find the ratio of the area of $ABKGH$ to the area of $ABCDEF$.

   *Proposed by: Jack Gao*

   **Answer:** $\boxed{\frac{23}{72}}$

   **Solution:** For a polygon $A_1A_2 \dots A_n$, we denote its area by $[A_1A_2 \dots A_n]$.




   Let the side length of the hexagon be $s$. Firstly, note that

$$[AFE] = \frac{1}{2}s^2 \sin 120^\circ = \frac{s^2\sqrt{3}}{4},$$

   and $[ABE] = \frac{1}{2}s\sqrt{3} \cdot s = 2[AFE]$. Thus,

$$\frac{[AFE]}{[ABCDEF]} = \frac{[AFE]}{2[AFE]+2[ABE]} = \frac{1}{6}$$

   and $\frac{[ABE]}{[ABCDEF]} = \frac{1}{3}$.

   Now, since $HA = \frac{1}{3}AF$ and $AG = \frac{1}{2}AE$, we have

$$[AHG] = \frac{1}{2}HA \cdot AG \sin \angle HAG = \frac{1}{6} \cdot \frac{1}{2}AF \cdot AE \sin \angle FAE = \frac{[AFE]}{6} = \frac{[ABCDEF]}{36}.$$

Similarly, $EG = \frac{1}{2}AE$ and $EK = \frac{1}{4}BE$, so

$$[KEG] = \frac{1}{2}EG \cdot EK \sin\angle KEG = \frac{1}{8} \cdot \frac{1}{2}AE \cdot BE \sin\angle AEB = \frac{[ABE]}{8},$$

so $[AGKB] = \frac{7}{8}[ABE] = \frac{7[ABCDEF]}{24}$. Now

$$\frac{[ABKGH]}{[ABCDEF]} = \frac{[AHG]}{[ABCDEF]} + \frac{[AGKB]}{[ABCDEF]} = \frac{1}{36} + \frac{7}{24} = \boxed{\frac{23}{72}}.$$

7. Calculate the number of ways to rearrange the letters in the name $WASHINGTON$ such that no vowel is adjacent to another vowel and all consonants are adjacent to another consonant.

   *Proposed by: Samuel Kretzschmar*

   **Answer:** $\boxed{151200}$

   **Solution:** We have three vowels and seven consonants (with one repeated letter). The letters can be written in the form

   $$\_V\_V\_V\_,$$

   where the _'s represent consonants and the V's represent vowels. Given the problem conditions, the middle blanks must contain at least 2 consonants and the end blanks cannot contain 1 consonant. The maximum number of consonant blocks is $\lfloor \frac{7}{2} \rfloor = 3$.

   Assume for now that all the consonants are indistinguishable and similary for the vowels; we can do casework as follows:

   - The first three gaps contain blocks of consonants: Automatically, each gap has at least 2 consonants, and the last consonant can be distributed among any of the three gaps, yielding 3 cases.
   - The last three gaps contain blocks of consonants: Symmetrically, this also gives us 3 cases.
   - The first and last gaps contain no consonants: The middle two gaps must sum to 7 total consonants, so we get pairs ranging from $(2, 5)$ to $(5, 2)$. This gives us 4 cases.

   In total, we have $3 + 3 + 4 = 10$ valid cases. For each of these cases, the vowels can be distributed in any order from left to right and the same thing occurs for the consonants. Our final answer is $10 \cdot 3! \cdot \frac{7!}{2!} = \boxed{151200}$.

8. Let $z = 2(\cos 40° + i \sin 40°)$, and let $\omega = z - z^7$. Determine the value of $|\omega|^2$.

   *Proposed by: Jack Gao*

   **Answer:** $\boxed{16644}$

   **Solution:** We will use the geometry of complex numbers to solve this. We know that the value $|z - z^7|$ represents the distance between the complex numbers $z$ and $z^7$. Since $z = 2(\cos 40° + i \sin 40°)$, $z^7 = 128(\cos 280° + i \sin 280°)$ by de Moivre's, so $|z| = 2$ and $|z^7| = 128$. On the complex plane, these two complex numbers and the origin form a triangle with side lengths 2 and 128, with the angle between them being $360° - (280° - 40°) = 120°$. Now, using the Law of Cosines,

   $$|\omega|^2 = 2^2 + 128^2 - 2 \cdot 2 \cdot 128 \cos 120°$$

$$= 4 + 16384 + 256$$
$$= 16644.$$

**Alternative Solution:** If $w = \frac{1}{2}z$, then $w^6 + w^{-6} = -1$, so

$$\begin{aligned}\omega\overline{\omega} &= (2w - 128w^7)(2w^{-1} - 128w^{-7}) \\ &= 4 + 128^2 - 256w^{-6} - 256w^6 \\ &= 4 + 256 + 128^2 = 16644.\end{aligned}$$

9. Square $ABCD$ has vertices $A$, $B$, and $C$ lying on the perimeter of rectangle $PQRS$ with $PQ = 7$ and $QR = 11$. Find the largest possible area of $ABCD$.

   *Proposed by: Samuel Kretzschmar*

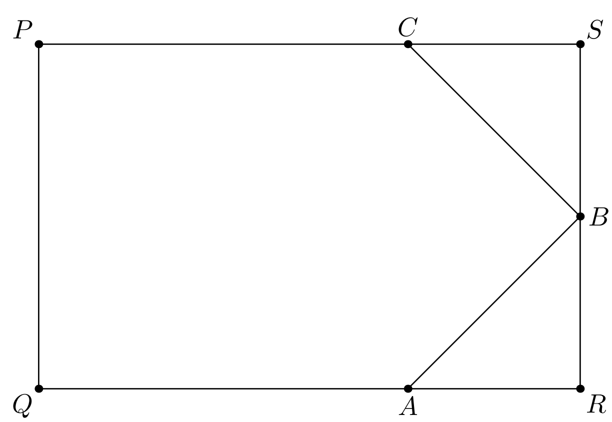
   **Answer:** $\boxed{65}$

   **Solution:** Place $A$ on the perimeter first. We have a few cases.

   If $B$ is on the same side of $PQRS$ as $A$ (and $A$, $B$ only lie on this side so that $A$ and $B$ are not equal to any of the vertices of $PQRS$; if they are equal to one of the vertices, then $A$ and $B$ are on adjacent sides, so they are covered in the below cases), then $C$ needs to lie on the opposite side; thus, the side length of the square is 7, in which case the area is $7^2 = 49$, or the side length of the square is 11, which is impossible.

   **Case 1:** $A$ is on a long side of $PQRS$, and $B$ is on an adjacent side.

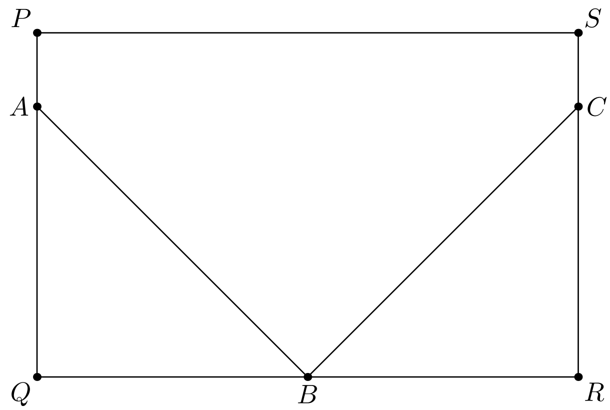
   We can WLOG $A \in QR$ and $B \in RS$.




   Observe that $\triangle ARB \cong \triangle BSC$. Let $0 \leq x = AR = BS \leq 7$; then, $7 - x = BR = SC$, so $AB = BC = \sqrt{x^2 + (7-x)^2}$, so the area of $ABCD$ is $x^2 + (7-x)^2 = 2x^2 - 14x + 49$. In the range $[0, 7]$, this is maximized at 0 and 7, which both give 49.

   **Case 2:** $A$ is on a short side of $PQRS$, and $B$ is on an adjacent side.

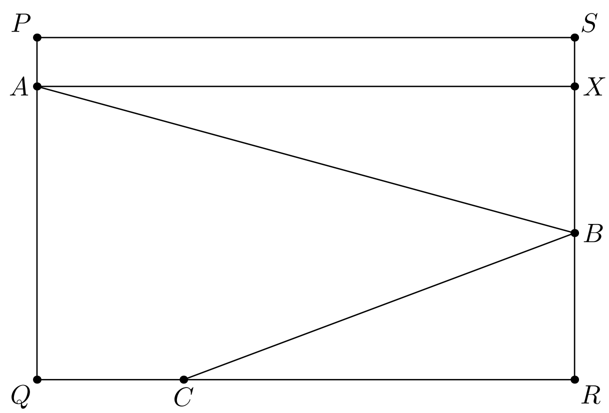
   We can WLOG $A \in PQ$, $B \in QR$, $C \in QR$




We have $\triangle ABQ \cong \triangle BCR$, so set $x = BR = AQ \leq 7$, then $11 - x = QB = CR \leq 7$, so $4 \leq x$. Thus, $AB = BC = \sqrt{x^2 + (11-x)^2}$, so the area of $ABCD$ is $x^2 + (11-x)^2 = 2x^2 - 22x + 121$; in the interval $x \in [4, 7]$, this is maximized at $x = 4$ or $x = 7$, which both give a value of 65.

**Case 3:** $A$ is on a short side of $PQRS$ and $B$ is on the opposite side.

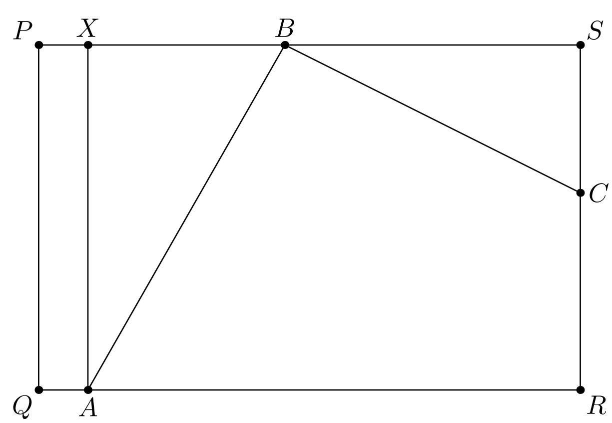
We can WLOG $A \in PQ$ and $B \in RS$.



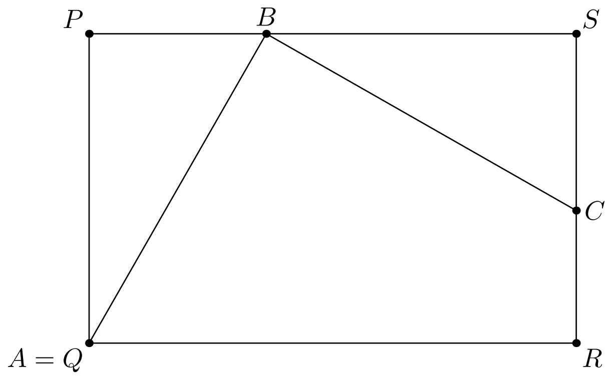

Let $X$ be the foot of the altitude from $A$ to $RS$. We have $\triangle ABX \cong BCR$, so $BR = AX = 11$, but $BR < RS = 7$, a contradiction. Thus there is no solution here.

**Case 4:** $A$ is on a long side of $PQRS$ and $B$ is on the opposite side.

We can WLOG $A \in QR$, $B \in SP$, and $C \in RS$.



Let $X$ be the foot from $A$ to $SP$. Then, $\triangle ABX \cong \triangle BCS$. Thus, $BS = AX = 7$. Thus, $AB = BC = \sqrt{AX^2 + XB^2} = \sqrt{49 + XB^2}$. Note that $XB \leq BP = 4$, so the area of the square is $49 + XB^2 \leq 65$. Equality occurs when $A = Q$, so the answer is $\boxed{65}$.




10. Integers $a$, $b$, $c$, and $d$ satisfy the equations $a^{\left(b^{(c^d)}\right)} = 27$ and $abcd = -54$. Find the sum of all possible values of $a + b + c + d$.

*Proposed by: Samuel Kretzschmar*

**Answer:** $\boxed{81}$

**Solution:** Let $x = a^{\left(b^{(c^d)}\right)} = 27$. Then for a prime $p$, $p \mid x$ if and only if $p \mid a$. Thus, $3 \mid a$ and $2 \nmid a$. Combining this with $a \mid -54$, we have $a \in \{3, -3, 9, -9, 27, -27\}$; however, if $a \in \{-3, -9, -27\}$, $b^{c^d}$ isn't a real number, so $a \in \{3, 9, 27\}$. However, if $a = 9$, $b^{c^d} = \frac{3}{2}$, which isn't possible for an integer base $b$ and rational exponent $c^d$, so $a = 3$ or $a = 27$. We now proceed by cases.

- If $a = 3$, then $b^{c^d} = 3$ and $bcd = -18$. As $3 \mid b^{c^d}$ and $2 \nmid b^{c^d}$, $b \in \{3, -3, 9, -9\}$, and since $c^d$ is real and $b^{c^d} = 3$, $b \in \{3, 9\}$.
  - If $b = 3$, then $c^d = 1$ and $cd = -6$. Checking factor pairs of $cd = -6$ gives the working pairs $(c, d) = (1, -6), (-1, 6)$ and the corresponding solutions $(a, b, c, d) = (3, 3, 1, -6), (3, 3, -1, 6)$ for sums of $3 + 3 + 1 - 6 = 1$ and $3 + 3 - 1 + 6 = 11$.
  - If $b = 9$, then $c^d = \frac{1}{2}$ and $cd = -2$. Checking factor pairs for $cd = -2$ yields a single working solution of $c = 2, d = -1$, with a corresponding solution $(a, b, c, d) = (3, 9, 2, -1)$ for a sum of $3 + 9 + 2 - 1 = 13$ (This case was the inspiration for this problem).
- If $a = 27$, then $b^{c^d} = 1$ and $bcd = -2$. Since $c^d \neq 0$, $b = 1$ or $b = -1$.
  - If $b = 1$, then $c^d$ can be anything and $cd = -2$. Checking factors of $-2$ gives the solutions solutions $(27, 1, -2, 1), (27, 1, -1, 2), (27, 1, 1, -2), (27, 1, 2, -1)$ for sums of $27 + 1 - 2 + 1 = 27$ and $27 + 1 - 1 + 2 = 29$.
  - If $b = -1$, then $c^d$ must be an even power and $cd = 2$. Checking factor pairs of 2 we see $1^2 = 1$, $2^1 = 2$, $(-1)^{-2} = 1$, and $(-2)^{-1} = -\frac{1}{2}$, so the only solution here is $(27, -1, 2, 1)$ for a sum of $27 - 1 + 2 + 1 = 29$ again.

Therefore, the sum of the different possible values for $a + b + c + d$ is $1 + 11 + 13 + 27 + 29 = \boxed{81}$.